PROPAGATION IN A SHIELDED MICROSLOT WITH A FERRITE SUBSTRATE

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Summary: By using the method of moments analysis is implemented for a shielded microslot, i.e., a slot line with a parallel ground plane. Accuracy in k_z of the order of 1% is obtained using matrices as small as 5 × 5.

Preliminaries: A problem closely related to that of the slotline [1,2,3] is the shielded microslot as depicted in Figure 1. For this analysis, region II will be filled with an ideal lossless ferrite, magnetized as indicated. All conductors are perfectly conducting the bifurcations are infinitely thin.

By assuming a time dependence of the form $e^{j\omega t}$ and a z dependence as $e^{-jk_z z}$ solutions are sought which have zero tangential electric fields at x = o,a in a homogeneous medium. This leads to fields that have an x variation as $\frac{\cos}{\sin}\{k_x X\}$ where $k_x = m\pi/a$ and a y variation as $\exp(jk_y y)$, with k_y to be determined. Four of Maxwell's six "curl equations" can now be solved for the y and z components in terms of the x components. Inserting these expressions into the remaining two curl equations and defining $s^2 = k_y^2 + k_z^2$ yields the familiar binomial equation in s^2 [4]. Thus, given m and assuming k_z determined, the fields can be represented in terms of four as yet undetermined parameters corresponding to the four values of $k_y = \pm(s_i^2 - k_z^2)$, i = 1,2. By applying the boundary conditions at x = o two of these unknowns can be eliminated, and the fields in region II can be written in terms of the unknowns for fixed m; similarly for region I, although with different k_y's.

If the bifurcations were not present, matching of the tangential fields at y = d would lead to four equations which could be satisfied (with proper eigenvalue, k_z) for any integer m. As it is, the condition that tangential $\underline{E}$ be continuous across the guide width is still valid and results in the elimination of two of the four unknown scalar coefficients, $A_{mi}(I, II)$. Since tangential $\underline{H}$ is only required to be continuous in the gap and the $\underline{E}_t$ field must be zero on the complimentary sectors, a single m mode is no longer sufficient. By summing over integer m and defining at y = d

$$J_z^E(x) \triangleq H_{IIx} - H_{Ix}\ , \quad J_z^H(x) \triangleq E_{IIx}$$

and

$$J_x^E(x) \triangleq H_{Iz} - H_{IIz}\ , \quad J_x^H(x) \triangleq E_{IIz}$$

the boundary conditions at y = d can be written,

$$\sum_{m=o}^{\infty} \{P_m(k_z) \int_{w_1}^{w_2} d\mu\, J_z^H(\mu)\cos\left(\frac{m\pi\mu}{a}\right) + Q_m(k_z) ⨍_o^a d\mu\, J_z^E(\mu)\sin\left(\frac{m\pi\mu}{a}\right)\}\cos\left(\frac{m\pi x}{a}\right) = J_x^E(x) \quad (1)$$

$$\sum_{m=o}^{\infty} \{R_m(k_z) \int_{w_1}^{w_2} d\mu\, J_z^H(\mu)\cos\left(\frac{m\pi\mu}{a}\right) + S_m(k_z) ⨍_o^a d\mu\, J_z^E(\mu)\sin\left(\frac{m\pi\mu}{a}\right)\}\sin\left(\frac{m\pi x}{a}\right) = J_x^H(x) \quad (2)$$

where $⨍_o^a d\mu = \int_o^{w_1} d\mu + \int_{w_2}^a d\mu$ and the P_m's, Q_m's, R_m's, and S_m's are known functions of the physical and electrical properties of the waveguide. Recognizing that it is only necessary to satisfy the above equations in the regions where the right hand sides are zero, we obtain a pair of coupled homogeneous Fredholm integral equations of the first kind, not unlike those of Zysman and Varon [6].

Solution: Previous work on edge singularities in ferrites [7] has shown that $J_z^E(\cdot)$ and $J_z^H(\cdot)$ are $O(r^{-1/2})$ while $J_x^E(\cdot)$ and $J_x^H(\cdot)$ are O(1) at the edges y = d, x = w_1, w_2. Thus all singularities are within the integrands in Eqs. (1) and (2) and will integrate to smooth functions. Solution to these integral equations will be obtained using the method of moments [8]. The expansion functions for $J_z^E(\cdot)$ and $J_z^H(\cdot)$ are chosen to contain the $x^{-1/2}$ singularity explicitly plus an infinite series of trigonometric functions. For simplicity the test functions are chosen to be the same as the expansion functions, i.e., Galerkin's method. All integrals thus obtained are readily evaluated on the computer. The integral equations now reduce to an infinite matrix eigenvalue problem with eigenvalue k_z and with the coefficients of the expansion functions serving as the eigenvector.

By judicious truncation the above infinite system is reduced to a finite one for obtaining practical, explicit solutions. Computational results showed that answers with accuracies of the order of 1% can be obtained using matrices as small as 5 × 5 for guides symmetrical about x = a/2

Results: The first guide studied is shown in the inset of Fig. 2. The substrate is dielectric ($\underline{\mu} = \mu_o \underline{I}$) with $\varepsilon_r = 9$. Since at dc the guide is a single conductor, a zero cutoff mode is not expected. The two lowest modes were found by evaluating a 7 × 7 matrix with m summed from 0 to 30. Included in the ω - β plot are the results of Mariani et al [9], for an infinite unshielded slotline. These points are presented only for comparison of the two different types of guides.

Another guide investigated is shown in the inset of Fig. 3. Only the even modes were investigated using a 5 × 5 matrix and summing m from 0 to 40 (m even.) The fundamental mode is almost reciprocal. This is not surprising if one pictures the structure as a ferrite-filled rectangular guide (region II) with a narrow slot in the broad wall. Since at these frequencies the slot is only a fraction of a wavelength wide one would expect the results to be very similar to the reciprocal, closed-guide modes. A rather dramatic characteristic of the next highest order even mode is the rate at which it goes into cutoff; there is no "pause" near the line $k_z = k_o$ as there is for a ferrite-loaded microstrip [7]. This may be interpreted as an indication that this mode is also largely confined to the ferrite region. Finally, there appears a mode which is unidirectional in that no corresponding mode was found in its image region. Such modes are of course not uncommon in ferrite-loaded structures.

Discussion: The model as represented here can be generalized by several straightforward changes. Lossy media can be introduced by making μ and ε complex in the computer program. The fundamental mode might be made a zero cutoff mode by replacing the electric walls at $x = o, a$ with magnetic ones. However, the most important change would probably be the inclusion of a region III below the ferrite slab. This would replace the electric wall at $y = o$ with impedance boundary conditions and the model would then represent a shielded form of the slotline.

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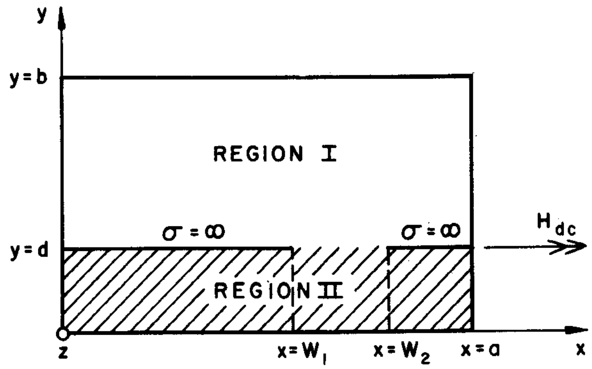

FIG. I SHIELDED SLOT LINE GEOMETRY.

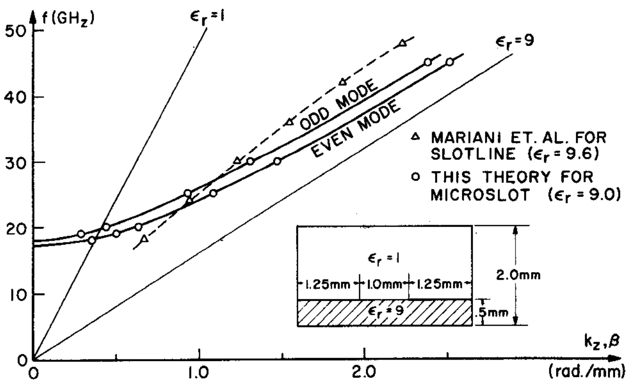

FIG. 2 $\omega - k_z$, (β), DIAGRAM FOR A WIDE SLOT ON A DIELECTRIC SLAB IN A SMALL GUIDE.

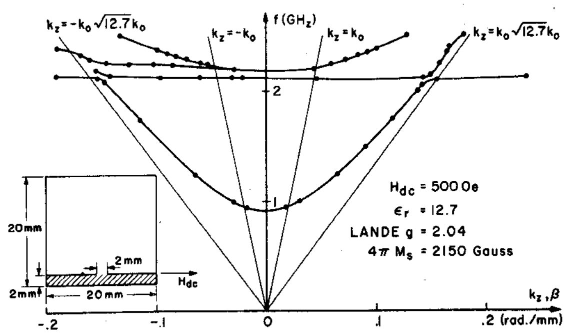

FIG. 3 $\omega - k_z, \beta$ DIAGRAM FOR A MICROSLOT ON A FERRITE (TRANS-TECH TTI-390) H_{dc} = 500 Oe.